An Alternate Algorithm to Evaluate the Reflected
Downward Flux Term for a Fast Forward Model

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at ITSC12 demonstrated the current algorithms for the attenuated reflected downward flux term did not work well for all the channels considered. In general, small biases existed only for high emissivities & low altitudes.

![Graph showing bias and standard deviation](image)

- $\triangle$ sec$\theta$=1, $\triangle$ sec$\theta$=1.25, $\triangle$ sec$\theta$=1.5, $\triangle$ sec$\theta$=1.75, $\triangle$ sec$\theta$=2
- HIRS, AIRS
affects land retrievals where emissivities may be considerably less than .9 and p_s < 800hPa

require a fast scheme that is acceptable for a wider range of emissivities and surface pressures
Top of the atmosphere (TOA) radiance is the sum of 3 terms:

- attenuated surface emissions
- attenuated atmospheric upward emissions
- attenuated reflected downward flux

\[
<\mathcal{R}_s(\theta, p_s) >= \left( \varepsilon B(T(p_s)) \mathcal{H}(\theta, p_s) \right) + \left( \int_0^p B(T) d\mathcal{H}(p, \theta) \right) + \left( r \mathcal{H}(\theta, p_s) F'(p_s) \right)
\]

- \(p\) - pressure
- \(\theta\) - satellite zenith angle
- \(B\) - Planck function
- \(\mathcal{H}\) - \(p\) to TOA transmittance
- \(\varepsilon\) - surface emissivity
- \(r\) - surface reflectivity
- \(F'\) - downward flux

Subscript 's' denotes a topographical or cloud top surface

\(\mathcal{R}, \mathcal{H}, B, \varepsilon\) and \(r\) are functions of wavenumber

\[
\cdot <f> = \int_{\Delta\tilde{\nu}} \phi(\tilde{\nu}) f(\tilde{\nu}) d\tilde{\nu}
\]

\(\phi\) - response function

Variables of the form \(<f>\) are evaluated using MSC’s Fast Line-By-Line (FLBL) radiative transfer model
Attenuated reflected downward flux (ARDF) term is approximated as

\[
\langle r \mathfrak{F}(\theta, p_s) F^i(p_s) \rangle \approx r \langle \mathfrak{F}(\theta, p_s) F^i(p_s) \rangle \approx r \langle \mathfrak{F}(\theta, p_s) \rangle \langle F^i(p_s) \rangle
\]

\[
r \langle \mathfrak{F}^\phi \rangle = \frac{1}{\pi} \left[ \sum_{k=1}^{\infty} \frac{\langle \mathfrak{F}^\phi_{k-1} \rangle - \langle \mathfrak{F}^\phi_k \rangle}{\langle \mathfrak{F}^\phi_{k-1} \rangle \langle \mathfrak{F}^\phi_k \rangle} \langle B_k \rangle \right] \langle \mathfrak{F}^\phi \rangle
\]

assume \( r \) is constant across \( \phi \)

- isotropic reflection for this work, ie \( r = (1-\varepsilon)/\pi \)

approximate \( F^i \) by replacing the angular integration of \( \mathfrak{F}^i \) with \( \mathfrak{F}(\phi) \), \( \text{sec}\phi \) is the diffusivity factor, usually set to 1.66

\( <a b> \) can be decomposed as \( <a> <b> \)

**RTTOV**  \( \mathfrak{F}(\phi) = \mathfrak{F}(\theta) \)  \( \text{Saunders, 1999} \)

**MSCFAST**  \( \mathfrak{F}(\phi) = \mathfrak{F}(1.66) \)  \( \text{Garand, 1999} \)

requires 2\(^{nd}\) pass of transmittance model
past experience tells us that \(<a b>\) can not be decomposed as \(<a> <b>\) (Turner, 2001)

test for reliability of the decomposition of the return transmittance and the downward flux using the FLBL, ie; how well does \(\delta BT = 0\)?

\[
\delta BT = BT\left(\left\{\mathcal{S}(\theta, p_s)F^i(p_s)\right\}\right) - BT\left(\left\{\mathcal{S}(\theta, p_s)\right\}\left\{F^i(p_s)\right\}\right)
\]

decomposition fares poorly
- plot the bias of \(\delta BT\) across 52 ECMWF profiles for \(\varepsilon=.98\) & \(\varepsilon=.7\)
- many channels exhibit large errors that increase with \(\theta, \varepsilon\) & \(p_s\)

if decomposition of \(\mathcal{S} F^i\) is unreliable, then further decomposition of \(F^i\) into [] is probably not reliable, thus new scheme must account for errors due to these decompositions
Sampling of biases across the 52 ECMWF profiles for \( \varepsilon = 0.98 \) (AIRS 889-1080)

\[
\delta BT = BT\left( \langle \mathcal{F}(\theta, p_s) F^\dagger(p_s) \rangle \right) - BT\left( \mathcal{F}(\theta, p_s) \langle F^\dagger(p_s) \rangle \right)
\]
Alternate Algorithm

Assume that for a given \((\theta, p_s)\) there exists a value \(\kappa\) such that replacing \(\mathcal{F}(\theta)\) with \(\mathcal{F}^\kappa(\theta)\) provides a good estimate of the ARDF term

\[
r \cdot \left< \mathcal{F}_\theta(\theta) \right> = \frac{1}{\pi} \left( \sum_{k=1}^{4} \frac{\left< \mathcal{F}^{\theta}_{k-1} \right> \kappa(p_s, \theta) - \left< \mathcal{F}^{\theta}_{k} \right> \kappa(p_s, \theta)}{\left< \mathcal{F}^{\theta}_{k-1} \right> \kappa(p_s, \theta) - \left< \mathcal{F}^{\theta}_{k} \right> \kappa(p_s, \theta)} \right) \left< B_{\theta} \right> \left< \mathcal{F}^{\theta}_{k} \kappa(p_s, \theta) \right>
\]

\(\kappa(\theta, p_s)\) is interpolated from a pre-determined look-up table.

Advantages:

- replaces the 2\(^{nd}\) pass of the fast transmittance model with a lookup table followed by an exponentiation should be faster
- accounts for decomposition of \(<S \hat{F}^\dagger>\)
- preserves current program structures hence, easier to implement
κ - Lookup Table Determination

- develop the basic fast transmittance model (ie $\varepsilon=1$)
- using the same atmospheres to develop the basic model, minimize

$$\left| \left( \mathcal{S}(\theta, p_s) F^{-1}(p_s) \right) - \{\mathcal{S}_\theta^0\} \left( \sum_{k=1}^{\delta} \frac{\{\mathcal{S}_{k-1}^\theta\} \chi(p_s, \theta) - \{\mathcal{S}_k^\theta\} \chi(p_s, \theta) - \{\tilde{B}_k\}}{\{\mathcal{S}_{k-1}^\theta\} \chi(p_s, \theta) \{\mathcal{S}_k^\theta\} \chi(p_s, \theta)} \right) \{\mathcal{S}_\theta^0\} \chi(p_s, \theta) \right| \leq \delta$$

for a set of $\kappa(\theta, p_s)$ for each atmosphere

- table entry is the average $\kappa(\theta, p_s)$ across the atmospheres

NOTE: $<f>$ - FLBL model, $\{f\}$ - fast model
Comparisons

- compare 3 modified forms of RTATOV (Saunders, 1999)
  - add extra levels at .005, .014, .037, 1048.51 & 1085 hPa
  - fast transmittance model coefficients determined from FLBL calculations using ECMWF 52 diverse profile set (AIRS inter-comparison)
  - 6 secants (1, 1.25, 1.5, 1.75, 2 & 2.25)

- $M_1, \, \varphi = \theta, \, \kappa = 1$ single pass thru’ fast transmittance model

- $M_2, \, \varphi = \theta, \, \kappa = 1$ two passes thru’ fast transmittance model

- $M_3, \, \varphi = \theta, \, \kappa = \kappa(\theta, p_s)$ single pass thru’ fast transmittance model followed by exponentiation of $\mathcal{F}(\theta)$
  - $\kappa(\theta, p_s)$ determined for 24 $p_s$ (223 to 1085hPa) and 6 secants (1, 1.25, 1.5, 1.75, 2 & 2.25)

- evaluate BT all 3 models & FLBL for
  - 24 surface pressures (223 to 1085hPa)
  - 21 emissivities (0 to 1), $r = 1/\pi$ to 0
  - 52 ECMWF atmospheres
  - 2378 AIRS channels

- compare bias and standard deviation (stdv) across 53 profiles of the difference,

\[
BT(\left< R_{surf} + R^\dagger + r \mathcal{F}(\theta, p_s) F^\dagger(p_s) \right> - BT(\left< R_{surf} \right> + \left< R^\dagger \right> + r \left< \mathcal{F}(\theta, p_s) \right> \left< F^\dagger(p_s) \right>))
\]
Fig: M1, M2 & M3 bias & stdv as a function of channel for sec $\theta = 1$, $\varepsilon = .7$ and $p_s = 1013$hPa

废气 M2 & M3 fare much better than M1

废气 not clear which performs better M2 or M3 wrt bias or stdv

废气 on average M3 is $\sim 1.25$ slower than M1 and M2 is $\sim 1.6$ slower than M1

废气 M3 faster than M2
Bias (left) & stdv (right) for channel 1018 (1007.86(cm⁻¹)) as a function of θ, ε & p_s

- strong θ dependency in M1, weaker in M2 & M3
- small region of low bias & stdv in M1 & M2
- M3 applicable over a wider range of ε & p_s
- M3 models the ARDF term very well in terms of bias
- stdv doesn’t improve using M3, but not any worse
Fig: Bias (left) & stdv (right) for channel 610 (851.8\,(cm^{-1})) as a function of $\theta, \varepsilon & p_s$

- strong $\theta$ dependency in M1, weaker in M2 & M3
- small region of low bias & stdv in M1
- M2 applicable over a wider range of $\varepsilon & p_s$
- example of when M2 better than M3
- some improvement in stdv over M1
Fig: More examples of the bias & stdv comparisons
Summary

- algorithm effects bias more than stdv
- both M2 & M3 are an improvement over M1
- M3 is faster than M2
- M2's &/or M3's stdv are generally no worse than M1's
- useful range of $\varepsilon$ and $p_s$ increased (ie manageable biases)
- ~65% of the channels perform as well or better than M2 with M3
Problems

- The bias vs channel curve contains many spikes
- Frequently M2 is better than M3 at these spikes

Fig: Upper box illustrates the bias curves for $\theta=0$, $\varepsilon=.6$ and $p_s=1013$ hPa (M1, M2, M3). The middle box is an enlargement of the upper box superimposed on a TOA total transmittance curve. The M1, M2, M3 values of {S} are marked by circles. The lower box is a further enlargement of the middle box with some AIRS spectral response functions superimposed.

- Problem channels are collocated with the core/near wing of H$_2$O spectral lines, these regions are very non-linear
- M3 needs more consideration prior to implementing M3
Conclusions

- The 2 pass transmittance model is preferable over the simple "reflection" model. Some tuning of the diffusivity factor may be required.

- The new algorithm is faster than current algorithms, but does not work for 100% of the channels. Ideally, we would like to use M3 exclusively, but need to "fix the spikes" first.

- Note that M3 does not depend on the relationship between r & ε; they can be independent of each other. Only require that they are constant over the response function.