Jacobian mapping between vertical coordinate systems in data assimilation
(ITSC-14 RTSP-WG action 2.1.1-c)

Atmospheric Science and Technology Directorate

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Introduction

Context:

- **Fast RTMs** for assimilation of radiances from nadir sounders often rely on regression based models evaluated on fixed pressure levels (e.g. RTTOV).

- Numerical prediction (e.g. **NWP**) models often use different vertical levels and a different vertical coordinate (e.g. η-hybrid).

- In this circumstance, **Jacobian mapping** from RTM to model coordinate is required in data assimilation (DA).
Data assimilation requires explicit pairing of the vertical interpolator and Jacobian mapping.

a) profile $x'$ on RTM levels $\leftrightarrow$ profile $x$ on model levels

$$ x'(p_i) = x'_i = s_i(x) = \sum_j W_{i,j} x_j \quad \text{or} \quad x' = W x $$

b) Jacobian mapping:

- model vertical coordinate $\leftrightarrow$ RTM vertical coordinate

$$ \left. \frac{\partial f}{\partial x_j} \right|_x = \sum_i \left. \frac{\partial f}{\partial x'_i} \right|_{x'} \frac{\partial x'_i}{\partial x_j} = \sum_i \left. \frac{\partial f}{\partial x'_i} \right|_{x'} W_{i,j} \quad \text{or} \quad h = W^T h' $$

The Jacobian mapping matrix is the adjoint $W^T$ of a linear forward model vertical interpolator matrix $W$ (or TLM of the interpolator).
Introduction

Identification of problem:

✓ Model levels not participating in forward interpolation (blind levels) lead to improper Jacobian mapping.

✓ Blind levels can result when the model vert. resolution is sufficiently higher than the RTM vert. resolution.

✓ Improper mapping heavily masked by vert. correlations of background covariances.
Introduction

Remainder of presentation:

✓ Identify an appropriate design for the vertical interpolator and its adjoint for use with fast RTMs in data assimilation when required (part 2 of ITSC-14 RTSC-WG action 2.1.1-c)

✓ Investigate sensitivity to choice of interpolator and representativeness quality of mapped Jacobians.
Interpolators for data assimilation:

- Nearest neighbour log-linear interpolator (operationally applied at EC for example)
- Proposed alternative: piecewise weighted averaging log-linear interpolator

\[
x'_i = \frac{\int_{i}^{i+1} w_i x \cdot d \ln p + \int_{i}^{i-1} w_i x \cdot d \ln p}{\int_{i}^{i+1} w_i \cdot d \ln p + \int_{i}^{i-1} w_i \cdot d \ln p}
\]

evaluated using the trapezoidal rule with weights \( w \) …
Weighting functions:
Nearest neighbour and piecewise weighted avg. interpolators

RTM levels

potential blind level
Mapping comparisons

Jacobian mappings via adjoint of:

- Nearest neighbour interpolator
- Proposed interpolator

Compared to

- Layer Thickness Scaling (LTS) interpolation for Jacobian mapping (no forward interpolator and adjoint pairing – not applicable to DA)
- RTM calculations on model levels (D.S. Turner)

using AMSU-A channels up to 14 and GFLBL (D.S. Turner) Jacobian calculations for AIRS (5) and HIRS (5) channels.

N.B.: LTS mapping method was used in Saunders et al. and Garand et al. RTM intercomparisons.
Mapping of AMSU-A Jacobians

RTTOV Ch. 13 (40)

Original

Proposed & LTS

CMAM levels

Nearest neighbour

Pressure (hPa)

Proposed

Pressure (hPa)

Weighting functions
Jacobian mappings for HIRS channel 12 for various (M,N)

Original from GFLBL

Mapped via Proposed LTS

Ref.: GFLBL

Profile relative error measure (%) over AIRS and HIRS channels and various (M,N):

71% with <5%
90% with <15%
for 17 280 cases
1D assimilation: Impact of vert. correl. & vert. interpolators

Sample temperature increments

Sample vert. correlation fns

NMC

6-hr diff.

NMC stats

6-hr diff.

nearest neighbour
3D-Var assimilation: Diagonal vert. correlation matrices

Average analysis profiles over 5 days at the equator

Nearest neighbour

Proposed
3D-Var assimilation: Impact of vert. correlation & vert. interpolators

CMAM-DA:
vertical correlation matrix from an ensemble perturbation approach (Yulia Nezlin)

~0.001 hPa
0.1 hPa
10 hPa
100 hPa
Surface

0.3
0.5
-0.2
3D-Var assimilation: Ensemble perturbation scheme vert. correlation matrices

Average profile differences over 5 days at the equator for both analyses and forecasts.

Curves show differences of temperatures obtained from using
- nearest neighbour
- proposed methods.
3D-Var assimilation: Impact on geopotential height (GEM model and NMC statistics: preliminary results)

For 6-hours forecasts in the tropical region.
Based on 12 days.

Nearest neighbour
Proposed
Summary and comments

✓ Proposed vertical interpolator satisfies Jacobian mapping requirements.

✓ P.S.: The forward vertical interpolator and its adjoint can account for surface pressure dependency of model coordinate when required.

✓ Level of benefit depends on vertical resolutions and width of vertical correlation functions.

✓ Stand-alone code to be made available shortly (contact: yves.rochon@ec.gc.ca and louis.garand@ec.gc.ca)

✓ Manuscript to QJRMS conditionally accepted.
Thank you!
LIST OF AIRS and HIRS CHANNELS FOR WHICH SIMULATIONS WERE PERFORMED. HWHM STANDS FOR THE HALF-WIDTH AT HALF-MAXIMUM OF THE JACOBIAN PROFILE

<table>
<thead>
<tr>
<th>Channel</th>
<th>Frequency (cm(^{-1}))</th>
<th>Pressure (hPa) at</th>
<th>Related atmospheric variable(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>peak</td>
<td>lower HWHM</td>
</tr>
<tr>
<td>AIRS 305</td>
<td>737.1</td>
<td>750</td>
<td>440</td>
</tr>
<tr>
<td>AIRS 453</td>
<td>793.1</td>
<td>900</td>
<td>670</td>
</tr>
<tr>
<td>AIRS 1090</td>
<td>1040.1</td>
<td>25</td>
<td>12</td>
</tr>
<tr>
<td>AIRS 1766</td>
<td>1544.3</td>
<td>340</td>
<td>260</td>
</tr>
<tr>
<td>AIRS 2197</td>
<td>2500.3</td>
<td>920</td>
<td>670</td>
</tr>
<tr>
<td>HIRS 1</td>
<td>668.9</td>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>HIRS 7</td>
<td>749.6</td>
<td>800</td>
<td>490</td>
</tr>
<tr>
<td>HIRS 8</td>
<td>898.7</td>
<td>820</td>
<td>620</td>
</tr>
<tr>
<td>HIRS 9</td>
<td>1028.3</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>HIRS 12</td>
<td>1481.0</td>
<td>400</td>
<td>280</td>
</tr>
</tbody>
</table>
Distribution of goodness of fit measure $m$ for four bounded ranges.

$$m = \sqrt{\frac{\sum_{i=1}^{N} (y_i - y_{i,\text{ref}})^2}{\sum_{i=1}^{N} (y_{i,\text{ref}})^2}} \times 100\%$$

17,280 cases
International TOVS Study Conference, 15th, ITSC-15, Maratea, Italy, 4-10 October 2006
Madison, WI, University of Wisconsin-Madison, Space Science and Engineering Center,