Accounting for Correlated Satellite Observation Error in NAVGEM

Bill Campbell and Liz Satterfield
Naval Research Laboratory, Monterey CA

ITSC-20
Oct 27 – Nov 3, 2015
Lake Geneva, WI, USA
Sources of Observation Error

1) Instrument error (usually, but not always, uncorrelated)
2) Mapping operator (H) error (interpolation, radiative transfer)
3) Pre-processing, quality control, and bias correction errors
4) Error of representation (sampling or scaling error), which can lead to correlated error:

<table>
<thead>
<tr>
<th>True Temperature in Model Space</th>
<th>Subgrid Scale Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>T=28°</td>
<td>T=38°</td>
</tr>
<tr>
<td>T=30°</td>
<td>T=44°</td>
</tr>
<tr>
<td>T=32°</td>
<td>T=53°</td>
</tr>
</tbody>
</table>
Current Practice

• Until recently, most operation DA systems assumed no correlations between observations at different levels or locations (i.e., a diagonal $R$)

• To compensate for observation errors that are actually correlated, one or more of the following is typically done:
  – Discard (“thin”) observations until the remaining ones are uncorrelated (Bergman and Bonner (1976), Liu and Rabier (2003))
  – Local averaging (“superobbing”) (Berger and Forsythe (2004))
  – Inflated the observation error variances (Stewart et al. (2008, 2013))

• Theoretical studies (e.g. Stewart et al., 2009) indicate that including even approximate correlation structures outperforms diagonal $R$ with variance inflation

• *In January, 2013, the Met Office went operational with a vertical observation error covariance submatrix for the IASI instrument, which showed forecast benefit in seasonal testing in both hemispheres (Weston et al. (2014))*
Several methods exist which can inform estimates of the background and/or observation error covariance matrices.

All methods have free parameters and make different assumptions; none are clearly superior to the others.

Knowledge of when and how each method may produce sub-optimal results is the subject of current research.

**Desroziers’ Method**
(Desroziers et al. 2005)

\[
\left\langle (\mathbf{O} - \bar{F})(\mathbf{O} - \bar{A})^T \right\rangle = \mathbf{R}
\]

\[
\left\langle (\bar{A} - \bar{F})(\mathbf{O} - \bar{F})^T \right\rangle = \mathbf{H} \mathbf{B} \mathbf{H}^T
\]

\[
\left\langle (\mathbf{O} - \bar{F})(\mathbf{O} - \bar{F})^T \right\rangle = \mathbf{R} + \mathbf{H} \mathbf{B} \mathbf{H}^T
\]

**Hollingsworth-Lönnberg Method**
(Hollingsworth and Lönnberg, 1986)

Assumes no spatially-correlated observation error

Extrapolate red curve to zero separation, and compare with innovation variance (purple dot)

Mean of ob minus forecast (O-F) covariances, binned by separation distance

**Observation Based Methods**
e.g. Oke and Sakov 2007

(b) 1° averaged mSLA field
Preconditioning is done with $B^{-1/2}$

Iteration is done on this problem. We need to invert $R$!
4DVar Dual Formulation

\[(HBH^T + \tilde{R})z = (y - Hx_b)\]

\[\tilde{R}^{-1/2} (HBH^T + \tilde{R})z = \tilde{R}^{-1/2} (y - Hx_b)\]

Change of variables

\[w = \tilde{R}^{1/2} z\]

\[z = \tilde{R}^{-1/2} w\]

\[R = \tilde{R}^{1/2} C \tilde{R}^{1/2}\]

\[\tilde{R} = \text{diag} \{\sigma_{i,j}\}\]

\[\tilde{R}^{-1/2} (HBH^T + \tilde{R}) \tilde{R}^{-1/2} (\tilde{R}^{1/2} z) = \tilde{R}^{-1/2} (y - Hx_b)\]

\[(\tilde{R}^{-1/2} HBH^T \tilde{R}^{-1/2} + I)w = \tilde{R}^{-1/2} (y - Hx_b)\]

Iteration is done on the partial step and then mapped back with \(BHT\).
Application to ATMS

Advanced Technology Microwave Sounder (ATMS)
13 temperature channels
9 moisture channels

Cross Track
Along Track
Desroziers’ method estimate of interchannel portion of observation error correlation matrix for ATMS.

Current observation error correlation matrix used for ATMS, and for ALL observations.
Practical Implementation: What about Convergence?

• The condition number of a matrix $X$ is defined by $\sigma_{\text{max}}(X)/\sigma_{\text{min}}(X)$, which is the ratio of the maximum singular value of $X$ to the minimum one. (Singular value == eigenvalue for symmetric $X$)

• Adding correlated error increases the condition number, slowing down convergence of the solver.

• We can control how long the solver takes by constructing an approximate matrix with any condition number we choose.

• How to improve conditioning:
  1. Preconditioning by multiplying by diagonal scaling matrices
  2. Increase the diagonal values (additively) of the matrix (e.g. Weston et al. (2014)).
  3. Find a positive definite approximation to the matrix by altering the eigenvalue spectrum (Ky-Fan $p$-$k$ norm).
ATMS Original and Reconditioned Eigenspectra

Ky-Fan and Additive Reconditioning

Log(Eigenvalue)
Practical Implementation: Cauchy Interlacing Theorem

What happens when radiance profiles are incomplete (i.e., at a given location, some channels are missing, usually due to failing QC checks)?

**Cauchy interlacing theorem**

Let $A$ be a symmetric $n \times n$ matrix. The $m \times m$ matrix $B$, where $m \leq n$, is called a *compression* of $A$ if there exists an orthogonal projection $P$ onto a subspace of dimension $m$ such that $P^*AP = B$. The Cauchy interlacing theorem states:

**Theorem.** If the eigenvalues of $A$ are $\alpha_1 \leq \ldots \leq \alpha_n$, and those of $B$ are $\beta_1 \leq \ldots \leq \beta_j \leq \ldots \leq \beta_m$, then for all $j < m + 1$,

$$\alpha_j \leq \beta_j \leq \alpha^{n-m+j}$$

Notice that, when $n - m = 1$, we have $\alpha_j \leq \beta_j \leq \alpha_{j+1}$, hence the name *interlacing* theorem.
## Experimental Design

<table>
<thead>
<tr>
<th>Experiment Name</th>
<th>95% CI</th>
<th>99% CI</th>
<th>99.99% CI</th>
<th>Mean Iter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>atid</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>56</td>
<td>Control run, no correlated error for ATMS or IASI, default diag(R)</td>
</tr>
<tr>
<td>atmasc018</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>68</td>
<td>Recondition Desrozier ATMS correlation matrix to 18, default diag(R)</td>
</tr>
<tr>
<td>iasic169</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>72</td>
<td>Recondition Desrozier IASI correlation matrix to 169, default diag(R)</td>
</tr>
<tr>
<td>atmiasi</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>78</td>
<td>Both of the above</td>
</tr>
<tr>
<td>Dzratmasc018</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>81</td>
<td>Same as atmasc018, Desrozier diag(R), moisture 1/2 compromise diag(R)</td>
</tr>
<tr>
<td>Dzriasic169</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>88</td>
<td>Same as iasic169, Desrozier diag(R), moisture 1/2 compromise diag(R)</td>
</tr>
<tr>
<td>Drsatmsiasi</td>
<td>10</td>
<td>4</td>
<td>4</td>
<td>104</td>
<td>Same as atmiasi, Desrozier diag(R), moisture 1/2 compromise diag(R)</td>
</tr>
<tr>
<td>Wesatmasc018</td>
<td>13</td>
<td>4</td>
<td>3</td>
<td>65</td>
<td>Same as Dzratmasc018, but uses Weston-style reconditioning</td>
</tr>
<tr>
<td>Wesiaisc169</td>
<td>12</td>
<td>9</td>
<td>2</td>
<td>84</td>
<td>Same as Dzriasic169, but uses Weston-style reconditioning</td>
</tr>
<tr>
<td>Wesboth</td>
<td>16</td>
<td>13</td>
<td>3</td>
<td>87</td>
<td>Same as Drsatmsiasi, but uses Weston-style reconditioning</td>
</tr>
</tbody>
</table>

### ATMS only

### IASI only

### ATMS & IASI
Wesboth (Proposed Scorecard*)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Level</th>
<th>Region</th>
<th>Variable</th>
<th>Lead Time</th>
<th>Metric</th>
<th>Weight</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Buoy</td>
<td>None</td>
<td>NH</td>
<td>Wind Speed</td>
<td>72</td>
<td>Mean Error</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Fixed Buoy</td>
<td>None</td>
<td>SH</td>
<td>Wind Speed</td>
<td>72</td>
<td>Mean Error</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Fixed Buoy</td>
<td>None</td>
<td>Tropics</td>
<td>Wind Speed</td>
<td>72</td>
<td>Mean Error</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>100.0</td>
<td>Global</td>
<td>Geopotential</td>
<td>72</td>
<td>RMSE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>250.0</td>
<td>Global</td>
<td>Air Temp</td>
<td>72</td>
<td>RMSE</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>250.0</td>
<td>Global</td>
<td>Wind</td>
<td>72</td>
<td>Vector RMSE</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>500.0</td>
<td>Global</td>
<td>Geopotential</td>
<td>72</td>
<td>RMSE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>850.0</td>
<td>Global</td>
<td>Air Temp</td>
<td>72</td>
<td>RMSE</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Radiosondes</td>
<td>850.0</td>
<td>Global</td>
<td>Wind</td>
<td>72</td>
<td>Vector RMSE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>200.0</td>
<td>NH</td>
<td>Wind</td>
<td>72</td>
<td>Vector RMSE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>200.0</td>
<td>Tropics</td>
<td>Wind</td>
<td>72</td>
<td>Vector RMSE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>500.0</td>
<td>NH</td>
<td>Geopotential</td>
<td>96</td>
<td>AC</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>500.0</td>
<td>SH</td>
<td>Geopotential</td>
<td>96</td>
<td>AC</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>850.0</td>
<td>NH</td>
<td>Wind</td>
<td>72</td>
<td>Vector RMSE</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>850.0</td>
<td>Tropics</td>
<td>Wind</td>
<td>72</td>
<td>Vector RMSE</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>1000.0</td>
<td>NH</td>
<td>Geopotential</td>
<td>96</td>
<td>AC</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>EC-Analyses</td>
<td>1000.0</td>
<td>SH</td>
<td>Geopotential</td>
<td>96</td>
<td>AC</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

*Same as FNMOC standard scorecard, with self-analysis replaced by ECMWF analysis, confidence level from 95% to 99%, no thresholding
Main Conclusions

- The Desroziers error covariance estimation methods can quantify correlated observation error
- Minimal changes can be made to the estimated error correlations to fit operational time constraints
- After accounting for correlations, reducing default variances improves forecasts
- Correctly accounting for correlated observation error in satellite data assimilation improves forecasts
- One must be careful comparing experiments using scorecards, especially those with thresholding.