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REPORT TO THE UNIVERSITY OF WISCONSIN

LAKE INVESTIGATIONS COMMITTEE

CONTRIBUTION TO THEORETICAL

LIMNOLOGY

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CONTRIBUTION TO THEORETICAL LIMNOLOGY

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Investigations of the movement of water masses in lakes, which are set in effect through various influences, and in connection with the thermal structure, still have not been carried out with that thoroughness which they deserve; for the entire aeration of lakes is important and decisive. However, in other respects they are important and interesting also. They can still serve us as models for large stream phenomena in broad spaces of the oceans which we cannot so simply comprehend, and in which a synoptic work encounters almost insurmountable difficulties. The theoretical concept of this movement, and comparison of theory with reality, permits us to establish how the effect of the external influences upon the water masses takes place; it permits us the determination of individual coefficients which govern these effects, numerical calculations, and, to be sure, with an essentially greater accuracy than is possible from experimental investigation in laboratories. So we gradually obtain an insight into the dynamics of a lake and thereby also an insight as to what phenomena we should expect in the large water masses of the oceans.

In the following sections, two questions are discussed which concern the influence of turbulent friction upon the motion of water masses in a lake. In the first section is shown what influence the turbulent friction possesses upon the individual oscillation of a lake. There results that, especially with shallow lakes, the turbulent friction expresses itself in a very substantial lengthening of the period, which must be shown absolutely in the length of the seiche period. In the second section the circulation is determined which is produced in a doubly stratified lake by means of a wind current in the longitudinal
direction of the lake. The position of the boundary surfaces, the position of
the current boundaries, strength and direction of the current in the upper layers,
shows a dependence on the wind direction and strength, and shows a picture of
the circulation in lakes which is a regular phenomenon.

Questions which are connected with other phenomena in lakes still appear
frequently. Their answer in simple cases does not run into large difficulties.
As a contribution to this field only the above mentioned will be given. The
others follow occasionally in other places.

1. Oscillation of a lake under the influence of turbulent friction.

In a lake of rectangular cross-section the X axis lies in the undisturbed
upper surface. X = 0 is one end, X = l is the other end of the lake. Z axis
positive upwards. The bottom of the lake Z = -h. If we designate with c the
vertical displacement of the upper surface compared to its undisturbed position,
with u the horizontal velocity and with μ the coefficient of turbulent friction
(assumed constant) in ft/sec., then the equations of motion and continuity read:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial z^2} = \frac{\rho}{\eta} \frac{\partial c}{\partial t} \]  

As periodic motion of the free surface, with a period \( T = \frac{2\pi}{\omega} \), we take the
expression

\[ c = \xi \exp \left( - \frac{\omega t}{2} \right) \exp \left( i \omega t \right) \quad 0 < x < l \]  

(13)
since with friction a decay of amplitude of the oscillation is to be expected.
To this displacement belongs the horizontal velocity

\[ u = v(z) \exp \left( - \frac{\omega t}{2} \right) \exp \left( i \omega t \right) \]  

(14)
where \( v(z) \) is only a function of \( z \). The expression (14) fulfills the boundary
conditions that for \( x = 0 \) and \( x = l \), \( u = 0 \); when \( x \) is \( \frac{\pi}{L} \).
(13) and (14) inserted in (1,) produce the differential equation for \( v(z) \):

\[
\frac{d^2 v}{dz^2} + \frac{1}{\mu} + \frac{\gamma - \alpha}{\mu} i \frac{d}{dz} - \frac{z}{\mu} i \gamma = 0
\]  

(15)

The general solution if

\[
\mu \alpha^2 = \beta i - \gamma
\]

is

\[
u = \frac{\beta e^{\alpha z}}{\beta - i \gamma} \int e^{-\alpha \xi} \sin \xi + e^{i \alpha \xi} \frac{e^{-\alpha \xi} + e^{i \alpha \xi}}{e^{-\alpha h} + e^{-\alpha h}} d\xi
\]  

(16)

For the determination of the constants \( C_1 \) and \( C_2 \) the boundary conditions serve:

\[
z = 0, \quad \frac{\partial u}{\partial z} = 0, \quad \text{and for } z = -h, u = 0.
\]

We obtain finally

\[
u = \frac{\beta e^{\alpha z}}{\beta - i \gamma} \int e^{-\alpha \xi} \sin \xi + e^{i \alpha \xi} \frac{e^{-\alpha \xi} + e^{i \alpha \xi}}{e^{-\alpha h} + e^{-\alpha h}} d\xi
\]

(17)

and from the equation of continuity (1,2) as an equation for \( \gamma \) and \( \sigma \) which is the damping and period of the oscillation, the expression

\[
\alpha^2 + \frac{\gamma \mu}{2} \frac{x^2}{\mu^2} \left[ 1 - \frac{\gamma \mu}{2} \right] = 0
\]

(18)

If the friction coefficient \( \mu = 0 \), then \( 2 \gamma = \infty \) and the expression in parentheses equals 1, so that \( (\gamma - \alpha)^2 + \frac{\gamma \mu}{2} \frac{x^2}{\mu^2} = 0 \). This shows \( y = 0 \) and \( T = \frac{2}{\gamma \mu} \), the well-known Heretian formula.

The discussion of the relation (19), where we now arrive, furnishes several difficulties if we set

\[
\beta \alpha = \rho + \rho^\prime i
\]

in which \( \rho \) and \( \rho^\prime \) are real numbers, then
\[ h^2 a^2 = (\eta^2 - \xi^2) + \frac{2}{\eta} \eta \xi \]

in comparison with (16), gives for the period of the free oscillation

\[ \lambda' = \frac{\mu}{h} \frac{2}{\eta} \xi \eta \]

and for the damping of (16).

\[ \gamma = \frac{\mu}{h} \frac{2}{(\xi^2 - \eta^2)} \]

The numerical values for \( \xi \) and \( \eta \) are found from the solution of (19). If we place (10) in (9), separate real and imaginary parts, which must both be equal to 0, we obtain two equations for the determination of \( \xi \) and \( \eta \). If we set

\[ \beta = \frac{\mu^2}{h^2} \]

then it reads

\[ [(\eta^2 - \eta^2)^2 - 4 \eta (\xi^2)(\eta^2 - \xi^2)](\kappa + \lambda \eta + \omega \eta \xi) + \frac{1}{h} \left[ (\eta^2 - \xi^2) - 4 \eta \xi \right] = 0 \]

\[ + \frac{1}{h} \left[ (\eta^2 - \xi^2) + 4 \eta \xi \right] \]

These equations for given values of \( \beta \) always have a definite pair of solutions which according to (111) determine the period and damping of the free oscillations.

From (112), equation

\[ \frac{1}{h} \left[ (\eta^2 - \xi^2)^2 - 4 \eta (\xi^2)(\eta^2 - \xi^2) \right] + \frac{1}{h^2} \left[ \eta^2 - \eta + \omega \xi \right] = 0 \]

may be derived. We see that if \( \eta = 0 \) the equation is satisfied. This value corresponds to an infinite period, therefore a boundary case. For \( \eta = 0 \) equation (113) gives two relationships from which we can eliminate \( \beta \) and as a relationship for \( \zeta' \)

\[ 6 \xi^2 + 4 \eta^2 \xi \eta \omega \xi + 5 \eta^2 - \lambda \eta^2 \xi = 0 \]

One root is \( \zeta' = 0 \), the second \( \zeta' = 1.1122 \) and to it belong \( \beta = 0.5370 \). For smaller values of \( \beta \) there are no real values for \( \eta \) and \( \xi \). For larger values of
\( \beta, \gamma, \) and \( \eta \) are to be determined from (113).

According to these expressions numerical values for \( \eta, \gamma, \) and \( \beta \) were determined. I remember similar values in a work by Proudman and Doodson, (Proc. London Math. Soc. Ser. 2, Vol. 24, Part 2, 1924). Actually it appeared to be a similar solution of the equation of motion without application to the problem indicated here. Its quantities \( \beta^2 \) and \( \gamma \) are concealed with the ones used here instead of \( \beta^2 \) stands \( \chi \). Therefore I have not calculated \( \beta^2, \gamma, \) and \( \xi \) from (113) but have used another method. According to another method values as high as \( \beta^2 = 1.1 \) (calculated after Proudman and Doodson) are given in a following table.

These authors also call attention that for large values the \( \beta^2 \gamma \) curve approaches an asymptote \( \beta^2 = \gamma + \frac{1}{\mu} \). The oscillation of the surface now follows from the equation

\[
\frac{\gamma}{\mu} \left( 1 + \frac{\mu^2 - \beta^2}{\beta^2} \right)^{\frac{1}{2}} = \beta^2 \gamma \left( 1 + \frac{\mu^2 - \beta^2}{\beta^2} \right)^{\frac{1}{2}}
\]

so the period through

\[
T = \frac{\mu \beta^2}{\gamma}
\]

is given.

<table>
<thead>
<tr>
<th>( \beta^2 )</th>
<th>( \gamma )</th>
<th>( \gamma \beta^2 )</th>
<th>( \gamma \beta^2 )</th>
<th>( \beta^2 \gamma )</th>
<th>( \beta^2 \gamma )</th>
<th>( \gamma \beta^2 )</th>
<th>( \beta^2 \gamma )</th>
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<td>0.5370</td>
<td>0.0</td>
<td>1.1222</td>
<td>1.237</td>
<td>0.000</td>
<td>0.0000</td>
<td>0.000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.4732</td>
<td>0.2</td>
<td>1.1307</td>
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<td>0.452</td>
<td>0.3109</td>
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<td>0.3376</td>
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<td>1.241</td>
<td>0.947</td>
<td>0.5502</td>
<td>1.827</td>
<td>81.7</td>
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<td>0.6</td>
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<td>1.520</td>
<td>0.7022</td>
<td>1.424</td>
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<td>1.253</td>
<td>2.202</td>
<td>0.7800</td>
<td>1.269</td>
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<td>3.435</td>
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<td>1.174</td>
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</tr>
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<td>0.02440</td>
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<td>0.00268</td>
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<td>3.1416</td>
<td>1.822</td>
<td>17.824</td>
<td>0.9233</td>
<td>1.083</td>
<td>8.3</td>
</tr>
</tbody>
</table>
According to the lerian formula ($\mu = 0$) the time of oscillation of the basin is 
$$T = \frac{z}{g h}$$
so that between $T$ and $T'$ the relation
$$T = \frac{T'}{2^{\beta \frac{z}{h} \epsilon}}$$
exists. This relationship permits immediately the visualization of the influence of turbulent friction, for we see since the denominator in ($l_{17}$) is always smaller than one, it increases the period of the free oscillation and, of course, to the relative amount $\frac{1}{2^{\beta \frac{z}{h} \epsilon}}$; percentually $\frac{100}{2^{\beta \frac{z}{h} \epsilon}}\%$.
These values I have also given in the above table.

For the determination of the period of free oscillation of a basin for a given turbulent friction coefficient $\beta$, we have to proceed in the following manner. We determine from ($l_{12}$), from $h$, and $l$ of the basin the quantity $\beta$.

From the tables, or better from a graphical representation which we get from the table we determine for this value of $\beta$ corresponding values of $\epsilon$ and $\eta$ and the quantity $2^{\beta \frac{z}{h} \epsilon} \eta$, whence ($l_{17}$) gives the period $T$.

The last column of the above table shows what sort of significant increases of the period can be produced through turbulent friction, so that ignoring the frictional influence does not appear to be justified, especially when $\beta$ is large, i.e. with large friction and a shallow basin. Just one example might clarify this. The Plattensee has a longitudinal axis of 77.2 km.; its depth along this axis is 3.8 m. ; from April to May about 4 m. ; from September to November 3.3 m. The mean depth with the parts of the bank that are flat is 2.98 meters. The period of the unimodal seiche is on the average 11.5 hrs.; in the months of high water about 10 hours, in the months of low water a little over 12 hours (E. v. Cholnoky "Limnology of Plattensees" Res. der wiss. Erforsch des Plattensees I. Bd.,3Teil, Wien 1897).
The herian formula gives for \( h = 400 \text{ cm} \), \( T^1 = 6.85 \text{ hours} \), significantly less than observed. The form of the cross-section cannot make so large a lengthening of the period, since it is very simple; on the other hand the narrowing at Tihany produces a lengthening of the period. However the increase from 6.85 to 11.5 hours is, in view of the small depth of the lake, certainly also to be attributed to turbulent friction. \( T^1/T \) gives 0.595 and this value of \( \beta^1 = \beta^2 \) corresponds to the above table \( \beta = .30 \). We find therefore with the given relations that \( \mu = 45 \text{ cm}^2/\text{sec.} \), a value which appears suitable for the slow movements with this long oscillation. Turbulence appears in this special case as a condition which can very essentially increase the oscillation period of a water mass.

Section II

Wind driven circulation in a double layered lake.

Figure 1 gives the position of the lake with rectangular cross-section in the selected coordinate system. On the upper surface of the lake a wind of constant strength blows in the longitudinal direction and causes a circulation in the top layer so that by means of friction the upper most layer of water is carried along in the direction of the wind. In the steady state it will finally cause a circulation in the upper and lower layers, which must satisfy the equations of motion as well as the equation of continuity.

\[
\begin{align*}
-\gamma \frac{\partial^2 \theta}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} &= 0, \\
-\frac{\rho}{\beta^1} \frac{\partial v}{\partial x} + \frac{\rho}{\beta^2} \frac{\partial v}{\partial y} &= 0, \\
+ \frac{\mu}{\beta^2} \frac{\partial v}{\partial y} &= 0.
\end{align*}
\] (21)

\[
\begin{align*}
\int_{-h_1}^{0} u \, dy &= 0, \\
\int_{-h_2}^{-h_1} u \, dy &= 0.
\end{align*}
\] (22)
Simultaneously the following boundary conditions are to be fulfilled:

$$\text{For } z = 0 \quad \frac{\partial u}{\partial y} = \frac{g}{\rho_1} w \sin \theta \quad x$$  \hspace{1cm} (2.3)

By means of the constant $\beta$ the tangential shearing stress of the wind on the water surface is determined.

$$\text{For } z = -h \quad \beta_1 \frac{\partial u_1}{\partial y} = \beta_2 \frac{\partial u_2}{\partial y} - \nu (u_1 - u_2) \quad \text{ (2.4)}$$

The effect of the upper layer upon the lower water masses results through the friction of the moving upper mass, in which case the friction is proportional to the velocity difference of both masses:

$$\text{For } z = -h_2 \quad u_2 = 0 \quad \text{ (2.5)}$$

At the bottom of the lake the water clings to the ground. The solution of the above equations must have the form:

$$u_2 = \frac{g}{\mu_2} \beta_2(y) \sin \theta \cdot x \quad \text{ (2.6)}$$

$V_1$ and $V_2$ are only functions of depth and determine the velocity distribution in the vertical. For this result the relationships are:

$$z = \frac{y}{2} z + \nu \frac{\rho}{2} \left( \frac{\beta_2}{2} + h_s \right) \left( \frac{\beta_2}{2} + \frac{2 h_1 + h_2}{3} \right) \quad \text{ (2.7)}$$

whereby

$$\beta = \frac{\nu}{\rho_2} \beta_1 = \frac{\beta_2 - \beta_1}{\beta_2} \beta_1 \quad \text{ (2.8)}$$
The boundary conditions at the upper surface, at the bottom of the lake, as well as the second equation of continuity (2) are thereby fulfilled. The other three boundary conditions serve for the determination of the unknowns, \( \xi_1^o \), \( \beta \), and \( C \), whereby then according to (2) \( \xi_2^o \) is also given. We find according to several calculations:

\[
C = \frac{W h}{2} - \frac{1}{2} \xi_1^o h + \beta \frac{1}{2} (h_2 - h_1) \frac{I}{\rho_2} \left( \frac{F_2}{2} + \frac{2 F_1}{F_2} \right) (2)
\]

Furthermore:

\[
\xi_1^o = \frac{W}{h_2} \left[ \frac{1}{h_2} + \frac{3}{2} \frac{F_2}{F_2 - h_2} \right] \quad \xi_2^o = \xi_1^o h_2 \left[ \frac{h_2}{h_2 - h_1} \right] (2.10)
\]

Through these relationships the problem appears solved; for all unknowns are expressed through the wind influence \( W \).

A special case is, that the upper fluid slips upon the lower one, the latter therefore is not influenced through friction; then condition (2) assumes the form:

\[
f(x) = -h, \quad \frac{\partial \xi_1^o}{\partial y} = \frac{\partial \xi_2^o}{\partial y} = 0
\]

the expressions are especially simple:

\[
\xi_1^o = \frac{W}{h_2}, \quad \xi_2^o = \frac{3}{2} \frac{h_2}{h_1}, \quad \beta = 0, \quad C = \frac{\beta}{\rho_2 - \beta}, \quad C_1 \quad C_2 = 0 (2.11)
\]

This means, that to be sure the water mass under the boundary surface remains at rest (to be expected, since of course no force works upon it), however, the boundary surface itself assumes a position which is preceded opposite to that of the upper surface by means of the wind. Its inclination is however, due to the density difference of the two types of water significantly increased. An example might explain this: \( \frac{v}{c} \) is the surface velocity of the water produced by the wind at \( x \) equals \( \frac{a}{2} \) (middle of the lake), we will assume it to be 10 cm. per second. The structure of the lake may be given to the
following numbers: \( h_1 = 5 \text{ m} \), \( h_2 = 20 \text{ m} \), \( \mu_1 = 100 \text{ cm}^2/\text{sec} \), \( \beta = 0.998 \)

\[ h_2 = 20 \text{ m} \], \( \mu_2 = 20 \text{ cm}^2/\text{sec} \), \( \beta_2 = 1.000 \), \( f = 0 \text{ km} \), \( h = 10^{-5} \text{ cm} \)

for \( \gamma = \frac{g}{2} \), \( \alpha = 0 \), \( \nu = 10 \text{ cm/sec} \)

for \( g \) we assume approximately 1,000 cm. per second^2.

Then results from the relationship (211).

\[ u = \frac{1}{1 - \left[ \frac{10^3 \gamma^2}{2} \right]} \left[ \frac{10^3 \gamma^2}{2} + \left( \frac{3 \gamma}{4} + 1 \right) c \right] \]

\[ m_{10} \]

<table>
<thead>
<tr>
<th>m</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>10.0</td>
<td>9.6</td>
<td>9.4</td>
<td>-2.6</td>
<td>-0.9</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

The velocity of the upper surface is zero in the depth of 2.12 meters, above that depth motion is in the direction of the wind, below that opposite the direction of the wind. \( u_2 \) remains continuously equal to 0, the lower layer is at rest. The slack water because of the wind amounts to almost 12 cm., the difference in the water stand on both ends of the lake is about 24 cm.; on the other hand the inclination of the boundary surface causes an elevation of the latter up to 6 meters over the horizontal position, therefore in 3.14 km. an increase of height of the boundary surface of 12 meters. Since the top layer itself is only 5 meters thick, in this extreme case by means of the wind such a transformation of the water masses results, that in the lee of the wind the lower water will attain the upper surface. Our above relations apply only as long as \( e \), \( g \), \( f \), \( h \), \( h_1 \), \( h_2 \), \( \gamma \), \( \alpha \), \( \beta \), and \( \beta_2 \), are small compared to \( h_2 \) and \( h_1 \). We see how strongly the wind can affect the horizontal structure of the water mass of the lake (about 6 to 7 bouefert).

For the case that the lower water layer is carried along by friction with the upper layer we will select the following example:

\( h_1 = 5 \text{ m} \), \( h_2 = 20 \text{ m} \), \( \gamma = 5 \text{ cm}^2/\text{sec} \), \( \beta = 0.998 \)

\[ h_2 = 20 \text{ m} \], \( \mu_1 = 20 \text{ cm}^2/\text{sec} \), \( \mu_2 = 10^{-5} \text{ cm} \), \( f = 0 \text{ km} \), \( h = 10 \text{ cm/sec} \)

Then result relationships (29) and (210).

\[ \gamma = 0.125 \text{ m} \]

\[ u = \frac{1}{1 - \left[ \frac{10^3 \gamma^2}{2} \right]} \left[ \frac{10^3 \gamma^2}{2} + \left( \frac{3 \gamma}{4} + 1 \right) c \right] \]

\[ \beta = 6.15 \text{ m} \]

\[ u = -4.175 \times 10^{-6} \left( \gamma + 10 \times 10^{-2} \right) \left( \gamma + 10 \times 10^{-2} \right) \]
The vertical distribution of velocities is given in the following table:

Upper layer:

<table>
<thead>
<tr>
<th>Depth in meters</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity in middle of lake (cm/sec)</td>
<td>10.0</td>
<td>4.64</td>
<td>0.34</td>
<td>2.62</td>
<td>4.32</td>
<td>4.78</td>
</tr>
</tbody>
</table>

Bottom layer:

<table>
<thead>
<tr>
<th>Depth in meters</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity in middle of lake (cm/sec)</td>
<td>3.13</td>
<td>2.34</td>
<td>1.63</td>
<td>1.60</td>
<td>0.46</td>
<td>0.00</td>
<td>0.67</td>
<td>1.00</td>
<td>1.00</td>
<td>0.67</td>
</tr>
</tbody>
</table>

It is inserted graphically in Figure 2. We see clearly the circulation of the upper layer with a changing current at a depth of 2.1 meters, the essentially weaker reversed circulation of the lower layer with a current change at 10 meters. At the boundary of the two types of water, the increase in velocity is 1.65 cm/sec. The strong inclination of the boundary surface against the slack water is due to the wind at the upper surface. The amount of this inclination deviates only a little from the amounts which were found in the preceding case of the slipping of the upper over the lower water. Also the velocity distribution in the upper layer is not essentially different. However in the first case the lower layer is at rest, in the second case it is in circulation. We notice how easily, through some friction, without the relations in the upper layer essentially changing, the lower layer likewise is activated into circulation.
The theory of wind-produced circulation here represented for enclosed water masses does not reveal anything that was not already known from experience and experiments in the laboratory. However, it permits us to comprehend the correlated phenomena quantitatively and to estimate the effect of the individual states correctly. Measurements on real lakes in this direction have been scarce, perhaps also they have been not systematic enough. It would be of profit first of all for the concept of the entire phenomenon, then also for the determination of the friction constants $\mu$, and $\gamma'$, for which our knowledge still is very sparse. However, in another respect are they of great interest. In many cases in a lake one initial layer is not present all by itself, but there are several layers more or less well developed over one another. Each layer has then its circulation always getting weaker from above towards below. In each layer by means of its circulation the type of water is made homogeneous in temperature and other properties even if first of all it was perhaps somewhat stratified. There takes place also in certain amounts a type of forced convection downward which can penetrate to the bottom. The correlation of several smaller layers with small wind influence is possible with the overcoming of the first difficulty. So the surface properties, (summer temperature, and others) move gradually into the dept's against stability, which to be sure is increased through the summer heating. This combined convection is one of the most important phenomena of all limnology.