INFRARED RADIATION MEASUREMENTS IN THE ATMOSPHERE

Annual Report
Contract CWB-9475

The research reported in this document has been sponsored by the Office of Meteorological Research, U.S. Weather Bureau, Director, Dr. Harry Wexler.
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ANNUAL REPORT ON RESEARCH PROGRESS AND PLANS, 1961

25 June 1961

U.S. Weather Bureau - University of Wisconsin
Department of Meteorology Project

Project Title: Differential Heating of an Air Column

Project Leader for U.S.W.B.: P.M. Kuhn, R. and D. Meteorologist, OMR

Principal Investigator for Dept. of Meteorology, University of
Wisconsin: Prof. V.E. Suomi

1. Present Status of Field of Investigation
   a. TIROS III Radiometersonde Program

   Presently Johnson Service Company of Milwaukee, Wisconsin
   is delivering specified Suomi-Kuhn (U.S. patent in progress)
   Radiometersondes to some twenty-five (25) U.S. Weather Bureau
   Radiosonde stations in Southeast United States. These units
   are the well known broad band balloon-borne radiometers which
   measure, synoptically, the upward, downward and net radiation
   currents as a function of height, radiators and temperature in
   the atmosphere.

   The radiometers will be used as a cross check on the
   selective IR detectors aboard Tiros III as was done for Tiros
   II. Close liaison between the Madison group and the Meteorological
   Satellite Laboratories, OMR, is maintained and computer data
   reduction at Madison is furnished the Satellite Laboratories.

   Some six hundred (600) processed Tiros II special, and
   NHRP radiometersonde flights are being used in the following
   studies.
b. **Selective Radiation Measurements**

Modifications to the broad response sensing elements of the Suomi-Kuhn radiometer are under way and will continue to provide a selectively sensitized detector in the ozone band, and the water vapor window in the atmosphere. Similar development is being carried out on independent, but simultaneous with the radiometersonde, measurements of ozone, water vapor and particulate matter throughout the atmosphere.

c. **Radiometersonde Estimates of Atmospheric Flux Emissivity**

This most interesting study appears to provide an in flight observed check on the various radiation charts and tables by checking atmospheric flux emissivity as a function of optical depth.

This study is part of P.M. Kuhn's Ph.D. thesis. It involves a solution of the radiative transfer equation for $\xi$, atmospheric flux emissivity, and $(d\xi/d\nu)$ the slope of the emissivity curve as a function of optical depth. It is centered upon this equation:

$$F_{\text{i}}\uparrow = \int_{0}^{U_T} \sigma T^4 \left( \frac{d\xi}{d\nu} \right) d\nu + (1 - \int_{0}^{U_T} \left( \frac{d\xi}{d\nu} \right) d\nu) \tilde{T}_5$$

(1)

where $F_{\text{i}}\uparrow$ is the upward IR radiation current

$T$ is the absolute temperature at a given level or mean for a given layer in the atmosphere

$\tilde{T}_5$ is the temperature of a homogeneous surface

$U$ is the corrected optical depth given by:
\[ du' = \frac{\bar{q}}{g} \frac{dp}{\rho_o} (\frac{P}{\rho_o})^m (\frac{T_o}{T})^n \]  

where  
\( \bar{q} \) is the mean specific humidity for a given layer in the atmosphere  
\( P \) is pressure  
\( \rho_o \) is surface pressure  
\( T_o \) is surface temperature  
\( g \) is acceleration of gravity

Numerical (machine) techniques are used to solve equation (1).

d. Comparison of Radiation Charts and Tables

By employing numerical means in the solution of equation (1) for \( F_{L\downarrow} \) or \( F_{L\uparrow} \), where \( F_{L\downarrow} \) is the downward propagating IR current, observed radiometersonde and computed radiation measurements may be compared. First results indicate the various radiation chart and table methods yield individual upward or downward radiation currents that are approximately 7 per cent lower than corresponding upward and downward observed IR currents. These are all carried out as functions of height in the atmosphere.

e. Computation of "Effective" High Level Water Vapor in the Atmosphere

Again by solving equation (1), numerically for \( du' \) or \( u' \), in this instance, we will apparently be able to compute actual "effective" water vapor far above the hygristor cut off point. Of course, we must strip out \( CO_2 \) and, possibly, ozone radiation contributions to obtain the "effective" water vapor content.
This will necessarily leave particulate matter as a radiator with the water vapor. However, scatter studies as noted in 1-b will we hope, help in restricting our results, eventually, to water vapor.

f. **Radiation Effects in the Tropopause**
   
   A further study on radiative effects at the tropopause is being undertaken to determine the role that IR flux plays in creating, maintaining, intensifying, or destroying the tropopause.

g. **National Hurricane Research Project Radiometersonde Flights**
   
   As was done belatedly in 1960, a new and earlier series of 200 radiometersonde penetrations of the atmosphere is planned for the 1961 hurricane season. This study will be worked out in cooperation with Dr. Herbert Riehl, Mr. Harry Hawkins, Mr. Ralph Higgs, OMR, The University of Wisconsin and the CHOS. Possible IR flux relations to hurricane development, maturation, and dissipation are being investigated.

2. **Progress Made During Fiscal Year 1961**
   
a. **Ground Check of the Accuracy of the Airborne Economical Radiometer**

   The manuscript of this article, accepted for publication in the August *Monthly Weather Review* is attached. The abstract will serve as a summary.

b. **Experimental Flight Verification of the Economical Net Radiometer**

   The manuscript, accepted for publication in a subsequent issue of the *Journal of Geophysical Research* is attached. The abstract will serve as a summary.
c. **On the Possibility of Thermal Circulations in the Balloon-Borne Radiometer**

   The manuscript of this article, submitted for publication to the *Journal of Meteorology* is attached. The abstract will serve as a summary.

d. **Tiros II Radiometersonde Program**

   All data has been reduced and is being used in studies l-c, l-d, l-e, l-f, and l-g.

e. **Radiometersonde-Sonde Estimates of Atmospheric Flux Emissivity**

   Numerical solutions of equation (1) have been applied and, as stated, first results indicate higher emissivities than are currently used in radiation charts and tables.

f. **Comparison of Radiation Charts and Tables**

   First results indicate an approximate 7 per cent increase in values of atmospheric-flux emissivity over the spectrum of optical depths.

g. **Computation of "Effective" High Level Water Vapor in the Atmosphere**

   Development of numerical solutions in underway. First tests are being made.

h. **Radiation Effects at the Tropopause**

   Study is being completed on Caribbean data with conclusions being presently drawn therefrom.

i. **National Hurricane Research Project Radiometersonde Flights**

   Data has been 75 per cent reduced. Studies are being planned for utilization of radiation data.
3. Future Investigation Plans

Due to the enormity of section 1, herein, future plans involve a carrying out of all listed studies under this section. In this regard, section 1 was a summary of current research as well as a summary of future plans.

4. Published Articles During Fiscal Year 1961


5. Articles Accepted Finally for Publication


6. Articles Submitted for Publication

ACCURACY OF THE AIRBORNE ECONOMICAL RADIOMETER

P.M. Kuhn

U.S. Weather Bureau, Department of Meteorology

The University of Wisconsin, Madison, Wis.

(Manuscript received March 30, 1961; revised May 31, 1961)

ABSTRACT

Further data are presented to indicate the accuracy of the airborne economical radiometer (frequently termed radiomertersonde when used in a modified radiosonde system) in the measurement of infrared radiation, in view of its recent widespread use. Three aspects are discussed. The first deals with a nocturnal ground comparison of the economical radiometer with a Suomi ventilated radiometer. The second covers an analysis of random errors in the net radiation obtained with the economical radiometer in the radiomertersonde system. And, finally, an experimental in-flight verification of the correctness of the conductivity term in the equations for the economical radiometer is discussed.
1. INSTRUMENT COMPARISON

A comparison of the Suomi ventilated radiometer (1) and the airborne economical radiometer (2, 3) was carried out 1.5 meters above homogeneous terrain during July 1960 at Madison, Wis. Consecutive nocturnal readings of both instruments taken at a sample rate of one each minute, resulted in 203 readings. The net radiation range observed was from 0.0091 to 0.1224 langley per minute. Recently Tanner et al. (4) discussed the accuracy of the economical radiometer in daytime measurements of total radiation.

The statistical results of the frequency distribution of the difference in net radiation observed with the airborne economical radiometer and the Suomi radiometer are presented in figure 1. The class interval for figure 1 is 0.0010 langley per minute. The mean difference between net radiation observed with the airborne radiometer and the ventilated net radiometer is 0.001059 langley per minute with an r.m.s. deviation of 0.001974 langley per minute.

The mean scale factor of the airborne radiometer to the ventilated radiometer is 1.01029 with an r.m.s. deviation of 0.02017. The reproducibility of the airborne radiometer readings during nighttime conditions in apparent from the results.

2. ERROR ANALYSIS

In a consideration of the effects of various possible random and systematic errors in the measurement of the downward- or upward-propagating radiation stream with the radiometersonde in balloon ascents, two sources of error are suggested by the general equation of Suomi and Kuhn (2, 3). They are random errors in conduction and black body radiation resulting from temperature measurement errors and systematic errors in conduction due
FIGURE 1.—Frequency distribution of the difference in net radiation observed with airborne economical radiometer (radiometersonde) and the Suomi ventilated radiometer.
to variations in the vertical dimensions of the instrument. We consider, first, errors in the temperature measuring system of the top and bottom sensing surfaces of the radiometer. The expression for net radiation obtained from equation (1) of Suomi, Staley, and Kuhn may be written:

\[
R_N = \sqrt{T_b^* - T_t^*} + 2 \left[ a \left( \frac{T_b + T_t}{2} \right) + b \right] \left[ T_b - T_t \right] - \left[ c \left( \frac{T_a + T_b}{2} \right) + d \right] \left[ T_a - T_b \right] + \left[ c \left( \frac{T_a + T_t}{2} \right) + d \right] \left[ T_a - T_t \right] + K \lambda \frac{d(T_b - T_t)}{dt} \tag{1}
\]

where \( R_N \) is net radiation in langley per minute; \( \sqrt{ } \) is 0.817 x 10^{-10} ly. min.^{-1} K.^{-1}; \( T_t \) is observed top sensor temperature, \( ^\circ K \); \( T_a \) is observed air sensor temperature, \( ^\circ K \); \( T_b \) is observed bottom sensor temperature, \( ^\circ K \); \( t \) is time in seconds; \( K \) is constant (1.45); and \( \lambda \) is constant product of effective specific heat and effective mass/cm² of the point-thermistor-Mylar sensor surface experimentally found from speed of response to be 7 x 10^{-3} cal/cm² deg.; \( a \) is 0.000005489 langley/minute\(^\circ C \); and \( d \) is 0.003608222 langley/minute.

The last term on the right side of equation (1) is identically zero when the radiometer is tested on the ground.

Differentiating equation (1) we obtain

\[
dR_N = \left[ 4 \sigma T_b^3 + (a + c) T_b + 2b + d \right] dT_b + \left[ -4 \sigma T_t^3 - (2a + c) T_t - 2b - d \right] dT_t \tag{2}
\]

The variance of the net radiation may be expressed as:

\[
VAR \left[ R_N \right] = k_1^2 VAR \left[ T_b \right] + k_2^2 VAR \left[ T_t \right] \tag{3}
\]

where \( k_1 \) is equal to the first term in brackets on the right side of equation (2) and \( k_2 \) is equal to the second bracketed term. \( T_b \) and \( T_t \) are independent of one another, but a standard error of 0.2\(^\circ C \) is assumed.
for each. The 0.2°C for the r.m.s. deviation was obtained after careful checks of the highest accuracy obtainable in reading the radiometersonde recorder chart and after extended measurements of the normal noise in the recorder system. This random error, of course, does not include the fixed bias error of the radiosonde receiver-recorder system, but the latter effect is of the second order.

Two cases were solved for equation (3): one in which the top and bottom radiometer temperatures were -70.0°C and -40.0°C, respectively, typical of the high troposphere or low stratosphere, and in the other in which these temperatures were 0.0°C and 15.0°C, typical of the lower troposphere. Computation gave a standard error of 0.0027 langley per minute in the first case and 0.0041 langley per minute in the second. The former produces an r.m.s. fluctuation of about 1 percent in the upper tropospheric net radiation value. We can also express the variance as the rate of temperature change in a 50-mb layer of the atmosphere (see equation (1) in Kuhn, Suomi and Darkow (5)). Since

$$\frac{\Delta T}{\Delta t} \left( ^{\circ}C/\text{day} \right) = \left[ \frac{1440}{c_p \Delta p} \right] \left[ R_{N, \text{top}} - R_{N, \text{surf}} \right]$$

the variance of the left and right sides of this equation becomes

$$\text{VAR} \left[ \frac{\Delta T}{\Delta t} \right] = \left[ \frac{1440}{c_p \Delta p} \right]^2 \text{VAR}(R_N)$$

where $\Delta T/\Delta t$ is the rate of temperature change due to the divergence of the net radiation in the vertical, $g$ is the acceleration of gravity, $c_p$ is the specific heat of air at constant pressure, $\Delta p$ is the pressure thickness of the layer considered, and $R_N$ is the net radiation.

Solving equation (5) for the two cases cited above yields standard errors in the daily rate of radiational temperature change for 50-mb atmospheric layers of 0.45°C and 0.68°C, respectively. For a 100-mb
FIGURE 2.—Cross section of the disc airborne economical radiometer (left) and standard airborne economical radiometer (right). Cross hatching indicates blackened aluminum sensing surface; $R_L$ is infrared radiation current; black dots are temperature sensors; subscripted T's refer to polyethylene, air, top, and bottom temperatures; upper case P's to polyethylene ventilation shields.
layer cases I and II give standard errors of 0.23°C and 0.34°C per day, respectively. These results, of course, involve no smoothing, which would tend to reduce the standard error. The 50-mb layer results give a median error of 0.50°C, for all ascent levels, to be compared with an earlier estimate of 0.60°C (5). The results described in this section are summarized in table 1.

3. CONDUCTIVITY TERM

In July 1958, a special balloon ascent was made carrying a "disc" type radiometersonde as well as the conventional radiometersonde. Details of this flight over Madison, Wis. were reported in (6). The importance of the "disc" radiometersonde in this simultaneous ascent becomes apparent from figure 2. Since it is exposed to the same streams of upward and downward propagating infrared radiation, it is clear that in the solution of the radiative balance equation for the "disc" radiometersonde there is no internal conduction term. Furthermore, in those instances when the upper and lower polyethylene (air) temperatures are equal to the disc temperature there can be no conduction between the outer polyethylene ventilation shield and the blackened "disc" surface.

If, then, the total radiation is measured both with the "disc" radiometer and the standard radiometersonde, the accuracy of the measurement of the internal conduction, top polyethylene to top sensing surface and bottom polyethylene to bottom sensing surface can be checked.

The total radiation measured by the "disc" radiometersonde is given by,

$$ R_{tot} = 2\sigma T_d^4 - 2 \left[ \alpha \left( T_a - T_d \right) + \beta \right] \left( T_a - T_d \right) + \left[ \frac{K \lambda d T_d}{a} \right] (6) $$
where $T_d$ is the observed "disc" temperature and the constants are defined as in equation (1). The total radiation measured by the standard radiometersonde is given by,

$$R_{tot} = \sqrt{(T_t + T_b)} - \left[ c \left( \frac{T_a + T_e}{2} \right) + d \right] [T_a - T_t]$$

$$- \left[ c \left( \frac{T_a + T_b}{2} \right) + d \right] [T_a - T_b] + \left[ K \lambda d (T_e + T_b) / dt \right]$$  (7)

where all terms are defined as in equation (1). Considering only those levels of the ascent where $T_a - T_d$ in equation (6) is equal to or less than $-1.2^\circ C$, we have computed $R_{tot}$ from equations (6) and (7) and compare them in table 2.

The data of the last column of table 2, the deviation of the disc from the standard radiometer, display a mean value of $-0.0117$ langley per minute. The r.m.s. deviation is $0.0206$ langley per minute. The mean deviation of the total radiation measured by the two instruments is approximately 2 percent. Since the same absorptivity for the blackened sensing surfaces is used in both instruments and since the same run polyethylene was used in both instruments it is evident that there is no lateral conduction loss. For if there were lateral losses the total radiation measured by the standard radiometersonde would be at best equal to that of the disc for a symmetric temperature profile (i.e., the air temperature is the mean of the observed top and bottom radiometer temperatures). Thus it is observed total radiation values springs in part from the baseline calibration and in part from the fact that $T_a - T_d$ in equation (6) was $-1.2^\circ C$.

In view of the importance assigned to atmospheric measurements of infrared radiation with the radiometersonde, it is believed that all indications to date suggest its continued use. Studies such as atmospheric flux emissivity, tropopause radiation, satellite radiation comparisons, and others are currently underway.
REFERENCES


LEGENDS

Figure 1. Frequency distribution of the difference in net radiation observed with airborne economical radiometer (radiometersonde) and the Suomi ventilated radiometer.

Figure 2. Cross section of the disc airborne economical radiometer (left) and standard airborne economical radiometer (right). Cross hatching indicates blackened aluminum sensing surface; $R_L$ is infrared radiation current; black dots are temperature sensors; subscripted $T$'s refer to polyethylene, air, top, and bottom temperatures; upper case $P$'s to polyethylene ventilation shields.
TABLE 1

Standard error of net radiation, and of the daily rate of radiational temperature change for 50-mb and 100-mb atmospheric layers.

<table>
<thead>
<tr>
<th></th>
<th>Net radiation</th>
<th>(\frac{\delta T}{\delta t}) (50-mb layer)</th>
<th>(\frac{\delta T}{\delta t}) (100-mb layer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0.0027 ly/min</td>
<td>0.45°C/day</td>
<td>0.23°C/day</td>
</tr>
<tr>
<td>Case II</td>
<td>0.0041 ly/min</td>
<td>0.68°C/day</td>
<td>0.34°C/day</td>
</tr>
</tbody>
</table>
Comparison of total radiation computed from equation (7)
for standard radiometersonde and from equation (6) for "disc"
radiometersonde

<table>
<thead>
<tr>
<th>Pressure (mb)</th>
<th>$R_{tot(stand.)}$</th>
<th>$R_{tot(disc)}$</th>
<th>$R_{tot(disc)} - R_{tot(stand.)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>1.130</td>
<td>1.165</td>
<td>-0.035</td>
</tr>
<tr>
<td>500</td>
<td>0.830</td>
<td>0.800</td>
<td>0.030</td>
</tr>
<tr>
<td>485</td>
<td>0.797</td>
<td>0.765</td>
<td>-0.032</td>
</tr>
<tr>
<td>350</td>
<td>0.600</td>
<td>0.566</td>
<td>-0.034</td>
</tr>
<tr>
<td>300</td>
<td>0.525</td>
<td>0.490</td>
<td>-0.035</td>
</tr>
<tr>
<td>250</td>
<td>0.440</td>
<td>0.433</td>
<td>-0.007</td>
</tr>
<tr>
<td>1100</td>
<td>0.340</td>
<td>0.338</td>
<td>-0.002</td>
</tr>
<tr>
<td>90</td>
<td>0.345</td>
<td>0.340</td>
<td>-0.005</td>
</tr>
<tr>
<td>82</td>
<td>0.346</td>
<td>0.345</td>
<td>-0.001</td>
</tr>
<tr>
<td>72</td>
<td>0.347</td>
<td>0.351</td>
<td>0.004</td>
</tr>
</tbody>
</table>
EXPERIMENTAL FLIGHT VERIFICATION OF THE ECONOMICAL NET RADIOMETER

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Abstract

In an experiment to verify the effect of convection and air conduction on the economical net radiometer, a flux plate type of radiometer, along with its required d-c amplifier, was designed, built and flown with a radiosonde. Two such flights were made. The net radiation values given by the flux plate agreed well with the values given by the economical net radiometer, showing that convection does not occur in nighttime use of the economical radiometer and that the air conductivity function being used is valid. An estimate is given of the precision of radiative cooling values which can be obtained from economical radiometer data.
Introduction

In the paragraphs that follow, Mr. Bushnell describes two test flights of balloon-borne radiometers flown in an attempt to obtain in situ system tests of the simple radiometer developed at the University of Wisconsin. This test is part of a continuing study to determine as completely as possible the performance of the instrument under all flight conditions.

This instrument, described in detail elsewhere [Suomi, and Kuhn 1958, Suomi, Staley and Kuhn 1958], is easily attached to the U.S. radiosonde. A considerable number of radiometer flights have now been made owing to the cooperation and help of the U.S. Weather Bureau. The principle usefulness of radiometer soundings is that they make possible measurements of the vertical derivative of the net radiation. This parameter is directly related to the cooling or heating of the atmosphere by long wave radiation. Areal differences in cooling rate can be identified as power available to drive the atmosphere.

The upward and downward streams of radiation are calculated from thermistor measured temperatures of the sensor surfaces and air temperature. Net radiation is calculated from the difference of these temperatures. The heating or cooling of the atmosphere, on the other hand, is obtained by observing the change of this difference with height. Clearly the heating or cooling of the atmosphere is derived from a second difference of the sensor temperatures. Even though the percentage error of each sensor temperature may be low, it is easy to show that the percentage error of the difference can be higher, the
degree of the degradation being inversely related to the magnitude of the difference. In a typical flight of the radiometersonde the difference between temperatures of the sensors near the earth's surface is about 10°, and it increases to 25-30° at 30 km. Now the size of the second difference and hence its percentage error depends on the size of the pressure interval over which the second difference is taken. Thus although the heating or cooling for a deep layer in the atmosphere can be measured to good accuracy, a thin layer of say 10 mb cannot be measured to good accuracy. Clearly, the real test of the instrument obtains in the stratosphere because there both the pressure intervals and the changes of radiation with height are small. If one is interested in studying departures from radiative equilibrium in the stratosphere, cooling rates as low as 0.1° per day could be important [Goody 1950]. Mr. Bushnell's results show that for a 100-mb pressure interval the standard error of the percent radiometersonde system in heating or cooling rates is 0.15° per day. The important point is that the limitation on accuracy is not at present determined by the radiometer instruments, but by the radiosonde telemetry system. The temperature sensitivity of the system becomes low at low temperatures. We have subsequently used a simple modification of the ground equipment whose purpose is to increase the low temperature scale factor and have reduced the uncertainty by a factor of 2. Therefore it is possible to determine cooling rates of .08°/day/100 mb.

When one considers the heating or cooling of deep layers in the atmosphere, such as the troposphere or the entire atmosphere, the precision of measurement increases accordingly. It is easy to detect
a cooling rate difference of only 0.02°/day. Cooling rates of 1.5°/day for arctic air and 3°/day for tropical air are typical, so accuracies of this measurement of better than 2 to 3 percent are possible.

Mr. Bushnell has included a table in which the magnitudes of the errors of other instruments are listed for comparison. These values were taken as those either mentioned or indicated in published figures of descriptions and in most cases are only mentioned incidentally and not as a result of a published error study. The reader is cautioned to keep this in mind.

Within the limitations outlined above, soundings of net radiation to good accuracy are possible and appear to be a very useful addition to the air temperature, pressure, and humidity measurements obtained in a typical radiosonde sounding.
I. Principle

Although the economical net radiometer has been shown to be correct on the ground [Tanner, Businger and Kuhn 1960; Kuhn 1961] some question has remained about its behavior high in the atmosphere. In particular, it was assumed that because the Grashof number [de Graaf and van der Held 1953] was always less than 1000, there was no convection in the thin air cells that protect the absorbing surfaces from ventilation. Also, the conductivity of the air in these cells was taken to be a certain function of temperature. These assumptions had not been verified experimentally for the radiometer in flight with its wide range of pressure and temperature.

Further, it has been suggested [Houghton 1958, Gergen 1958] that because the calculation of net radiation involves the difference of two numbers, the economical net radiometer may suffer from large errors which would make it almost useless for finding radiative cooling. Although Kuhn, Suomi, and Darkow [1959] indicate that this is not the case, they do not give experimental data.

Accordingly, we have done the experiment reported here, consisting of two radiosonde flights each carrying an economical net radiometer and a flux plate radiometer. We adapted a ventilated flux plate radiometer [Suomi, Franssela, and Islitzer, 1954] to balloon flight but did not use ventilation because of the power required and because ventilation would change with air density anyway. Instead, we used polyethylene convection shields, the same, in fact as we used on the economical radiometer. The flux plate radiometer is therefore the same as the economical radiometer in that it has two
blackened surfaces and polyethylene convection shields. In both, radiative energy is absorbed and then disposed of by reradiation and conduction. The difference in the two lies in the relative amounts of reradiation and conduction. In the flux plate radiometer typically 99.5 per cent of the net radiation energy is conducted through the glass flux plate, only .2 per cent is conducted away through the air cells, and .3 per cent is reradiated. In contrast to this, in the economical radiometer on the order of 50 per cent is conducted through the air cells and 50 per cent is reradiated. With the extreme difference, the two radiometers, in principle, should still agree in measured values of radiation. This agreement is the verification sought in the experiment. The flux plate develops a maximum temperature difference from top to bottom of only .3°C so that large temperature gradients cannot occur in the adjacent air cells formed by the shields. The data from the flights show that at times the temperature gradients were actually zero. Zero or not, we are assured that there is no significant air conduction and that the Grashof number in these cells is far smaller than in the economical radiometer.

The flux plate gives a recorder deflection essentially proportional to net radiation so, for it, there is no problem of the difference of large numbers.

In addition, as the flux plate radiometer has a time constant of .8 second, by using it we can check the corrections made for the 40 second time constant of the economical radiometer.
II. Structure of the Sondes

The flux plates, made of 68 turns of constantan wire wound on glass microscope slides and copper plated to make thermopiles, were painted black with the same paint used on the economical radiometers.

For the first one flown, the polyethylene convection shield consisted of four layers spaced 1/4 inch apart. The flux plate was mounted horizontally in the center cell as shown in the sketch in Figure 2. As polyethylene is transparent, one could see through the radiometer structure everywhere except through the plate itself and through the wood supports. With this arrangement radiation entering one face of the radiometer could pass through to the other side and be reflected by the polyethylene to the wrong face of the flux plate—a distinct disadvantage. After finding on the first flight that the temperature gradients in the air cells were indeed as small as we expected, we changed the design for the second radiometer in two ways. We used only one polyethylene sheet on each side of the radiometer mounting, and we put a black, opaque film in the plane of the flux plate so that no radiation could reach the wrong side of the plate. This radiometer is shown in Figure 1.

The 0 to 700 microvolt signal from the flux plate was telemetered to the ground and recorded by a standard radiosonde receiver. In this telemeter the flux plate signal was amplified with a chopper type d-c amplifier whose output controlled the frequency of the radiosonde blocking oscillator. The system was similar to that described by Suomi and Barrett [1952]. The flux plate signal, along with a reference voltage from a mercury cell, flux plate temperature,
air temperature, temperatures from the economical radiometer and monitoring signals, was selected in sequence by a motor-driven switch once every 30 seconds during flight.

The complete radiometersonde, consisting of a radiosonde, an amplifier, two radiometers, and all batteries, weighed 4200 grams. The flux plate radiometer itself weighed only 35 grams.

Now, because the thermal contact between the thermocouples and the glass plate is questionable, the flux plate scale factor cannot be obtained from its dimensions. It must be found by calibration. On the other hand, if the assumptions are correct, the scale factor of the economical radiometer is known without comparison with a standard instrument. In view of this, we have made the comparison between the two radiometers by calculating the scale factor of the flux plate for each 30 seconds of flight, using the economical radiometer as the standard. If the assumptions are not correct, this scale factor will, as a consequence, change with height.

The following items were included in the calculation of net radiation from the telemeter record of flux plate signals: response produced by the reference voltage, linearity correction to the telemeter, change of thermocouple emf with temperature of the flux plate (17 per cent change during a flight), change of thermal conductivity of glass with temperature, change of conductivity of air with temperature. The transmissivity and reflectivity of polyethylene and the absorptivity of the black paint, which are known for ground level conditions [Suomi, Staley and Kuhn, 1959; Tanner, Businger and Kuhn, 1960], were assumed not to change with height. For both radiometers the conductivity of air was taken to be

$$(348.4 + 1.06T) \times 10^{-5} \text{ cal/(cm min°C)}$$
where $T$ is the mean Celsius temperature of the air in a cell.

It should be noted that, to ensure having reliable data from the flux-plate-amplifier system, we used four of the nine segments on the motor-driven switch for monitoring. One segment connected the reference voltage to the input of the amplifier, another put a short circuit on the input, the third gave the temperature in the amplifier box and the fourth gave the filament voltage. We used these signals in accounting for voltage drift, in addition to using them to observe the condition of the sonde.

As the economical radiometer, in airborne use, contains standard U.S. Weather Bureau thermistors, we telemetered its temperatures in the conventional manner, just like air temperature.

III. Flights and Results

We made two night flights, both launched by the U.S. Weather Bureau radiosonde station at Green Bay, Wisconsin. In Figure 2, which shows the results for both flights, each point comes from a separate reading of the record; no smoothing has been applied. The values shown for the flux plate were obtained by using the mean scale factor for the whole flight. As can be seen, no large discrepancies appeared. However at the top of the April 22 flight the flux plate value is about 4 per cent below the economical radiometer value. This difference, caused by reflection from the polyethylene in the manner discussed above, did not appear in the second radiometer having the opaque film in the middle.

The fluctuations of the economical radiometer values are more or less the same over the whole of each flight and are larger than those
of the flux plate radiometer. They show the effects of using the difference of two temperatures and of errors in compensating for the long time constant. For the August 16 flight the rms fluctuation of the scale factor was 1.4 per cent.

IV. Estimate of the Errors of Cooling Values

From these data we can estimate the error of radiative cooling values obtained from economical radiometers. First we estimate the standard error of economical radiometer net radiation values as follows. The difference of the two radiation values at any one time in a flight is the combined error of both radiometers. This difference has a standard deviation of .0044 cal/(cm²·min). As the errors of the two radiometers are independent, the variance of this difference is the sum of the variance for the flux plate radiometer and the variance for the economical radiometer. Using a conservative estimate, based on the data, that the variance of the economical radiometer values is twice that of the flux plate values, the standard error for the economical radiometer is then .817 times the combined error. Thus we estimate the standard error of a net radiation value from an economical radiometer to be approximately .0036 cal/cm²·min), including the errors introduced by the calculations used to compensate for the time constant.

The rate of radiative temperature change is given by \( \frac{dT}{dt} = -\frac{g}{C_p} \frac{\Delta R_N}{A_p} \). The precision of \( \frac{dT}{dt} \) is proportional to the precision of \( \Delta R_N \) and inversely proportional to \( \Delta p \). \( \Delta R_N \), the difference of \( R_N \) at the top of a layer and \( R_N \) at the bottom, involves two measurements of \( R_N \) but, because their errors are independent, the standard error of \( \Delta R_N \) is \( \sqrt{2} \) times the standard error of net radiation.
To illustrate these errors, we consider two cases, a 50 mb layer and a 100 mb layer. For these two we find the errors of $\frac{dT}{dz}$ given in the first two lines of Table 1. In getting these values no use has been made of any kind of smoothing. Much better results can be had by combining the information in adjacent measurements in a flight. For example, if numerical smoothing is applied to the net radiation data so that the information from four points yields one smoothed point, then, because the four measurements are independent, the error of the cooling values is cut in half, as is shown in the last two lines of Table 1. This precision, twice as good as that given by Kuhn, Suomi, and Darkow [1959], appears to be routinely obtainable in nighttime soundings where the record is carefully read.

V. Conclusion

No gross differences appeared between the two radiometers. The differences which did appear are small and random. Therefore within the accuracy of these measurements, convection does not occur in nighttime use of the economical radiometer and the air conductivity function given above is valid. Radiation values obtained from the economical radiometer are not biased by large changes with height but are contaminated mainly by random fluctuations. The necessity of using the difference of two numbers in reducing economical radiometer data does not appear to cause serious errors. In spite of its fast response, the flux plate gave a smoother sounding than the economical radiometer gave. From this we conclude that the long time constant of the economical radiometer does not cause radiation structure to be missed.
The soundings of Pohl [1957], Aagard [1960] and Penn and Weickmann [1960] have a different character than ours, showing more and larger changes between heating and cooling. The comparison of the radiometers in this experiment shows that the economical radiometer would have indicated such extremes if they had existed at the times of our flights. Most other soundings made by economical radiometers likewise do not show such a number of changes between layers of large heating and cooling. Although the instruments have not been experimentally compared it would seem that the differences are a matter of the accuracy of the measuring systems and the smoothing used. In this connection Table 2 shows the precision of the measurements reported by these authors.

Acknowledgments. I wish to thank the staff of the U.S. Weather Bureau Airport Station at Green Bay, Wisconsin, for conducting the two soundings. P.M. Kuhn assisted in this experiment which was supported by the Office of Meteorological Research, U.S. Weather Bureau.
References


### Table 1  Precision of Radiative Cooling Values

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### Table 2  Precision of Net Radiation Values

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Figure 1. Flux plate radiometer used on the second flight.

Figure 2. Comparison of net radiation measurements made by the economical radiometer and the flux plate radiometer.
Fig. 1. FLUX PLATE RADIOMETER
ON THE POSSIBILITY OF THERMAL CIRCULATIONS
IN THE BALLOON-BORNE RADIOMETER

P.M. Kuhn
U.S. Weather Bureau, The University of Wisconsin

ABSTRACT

A theoretical and experimental determination of the possible strength of "thermal slope" circulation within the air cells of a tilted economical radiometer is presented. This is important in that the equations for the economical radiometer of Suomi and Kuhn assume no convection, which is correct for horizontal orientation. However, for tilted exposure, thermal slope winds could result in convection. Evidence shows that such slope winds do not produce convection.
TABLE OF SYMBOLS

(In order of appearance in text)

(All units in c.g.s.)

$$T = \text{Temperature in degrees Centigrade}$$

$$t = \text{Time in seconds}$$

$$U_S = \text{Up or downslope "thermal" wind}$$

Heating, $$\Theta > 0$$, results in $$\Delta T > 0$$ and upslope flow.

Cooling, $$\Theta < 0$$, results in $$\Delta T < 0$$ and downslope flow when

$$\Theta \bigg/ \bigg( \frac{\Delta T}{\Delta Z} \bigg) \bigg/ \sqrt{T_o} \ll$$

$$T_o = \text{Various isotherms}$$

$$\alpha = \text{Slope in inclination}$$

$$S = \text{Direction parallel to slope}$$

$$\gamma = \bigg[ \frac{\Delta T}{\Delta Z} \bigg] \bigg/ \sqrt{T_o}$$

$$T_o = \text{Surface temperature}$$

$$Z = \text{Distance above surface}$$

$$L = \text{Characteristic length, height of maximum thermal wind (cm).}$$

$$\mu = \text{Dynamic viscosity}$$

$$Pr = \text{Prandtl number} = \frac{\mu}{\gamma}$$

$$C_p = \text{Specific heat of air}$$

$$\lambda = \text{Thermal conductivity}$$

$$Q_o = \text{Surface density}$$

$$\Delta T_o = \text{Surface temperature disturbance}$$

$$\Theta_o = \text{Heating rates at slope surface forcing function}$$

$$U^* = \text{Characteristic slope wind speed}$$

$$g = \text{Acceleration of gravity}$$

$$D^* = \text{Displacement thickness of thermal slope flow}$$

$$U_{max} = \text{Maximum thermal slope wind speed}$$
THERMAL SLOPE CIRCULATIONS IN THE ECONOMICAL RADIOMETER

INTRODUCTION

The construction of the economical radiometer described by Suomi and Kuhn [1958] is such that in its airborne use the possibility of buoyancy forces produced by heat exchange at its inner sloping surface exists. Since the instrument is suspended by a long cord from the radiosonde balloon in flight [Kuhn and Suomi, 1960] there is, of course, a slow pendulum effect on the radiometer with a period of approximately 4-1/2 seconds during ascent.

Previous analyses of the presence or absence of convection phenomena in enclosed plane air layers do not involve the forcing function of heat transfer but rather consider only the temperature gradient from wall to wall. This study has its basis in the existence of heat exchange along a sloping boundary as the main variable. This permits evaluation of the Rayleigh module and the Reynolds module as criterion for convection.

THERMAL SLOPE CIRCULATION

The purpose of this study to determine the strength of "thermal slope winds" within the upper and lower air cells of the radiometer arising because of in-flight tilt effects and thereby determine the existence or non-existence of convection within the air cells. Since the radiometer solution [Suomi-Kuhn 1958] assumes no cellular convection between parallel planes we must also establish the non-existence of convective circulation produced by a sloping radiometer surface. In view of the long period of the oscillation of the radiometer we may assume, in our discussion, steady motion of a compressible viscous fluid (air) directly produced by heat exchange at a sloping boundary to maintain equilibrium between heat
diffusion and advection of heat. This treatment has been developed by H. Lettau, [1961] following the lines of Prandtl's [1942, 1952] studies. In the first instance we assume no upper boundary, and in the second we impose the restraint of a bounded upper polyethylene "cap" to our layer. In both instances the blackened lower sensing surface boundary is of infinite extent.

Heat exchange at a sloping boundary requires heat conduction normal to the radiometer. In steady states, which we have assumed, the process of heat diffusion is in equilibrium with heat advection produced by buoyancy-induced fluid flow parallel to the slope (of the radiometer sensing surface). For \( \frac{dT}{dt} = 0 \)

\[
\frac{dT}{dt} = U_s \frac{\partial T}{\partial s} \tag{1}
\]

It is obvious in (1) that with a constant \( \frac{dT}{dt} \) the lesser the slope the greater the "s" component of the velocity. This is also evident in figure 1.

Lettau's 1961 development parallel to that of Prandtl [1942, 1952] solves four equations, that of mass, momentum and heat continuity, and the equation of state for the four unknowns \( U_s, p, \rho, \) and \( T \). He expresses \( U_s \) as a function of both \( T \) and \( \rho \). The only concern is with the magnitude of the velocity component, \( U_s \), parallel to the slope. For it is this velocity, proportional to the slope and the magnitude of the heat exchange that could produce convection.
The basic assumptions in the solution may be summarized briefly, for clarity, as follows:

1) Gravity is the only external force.
2) Motion is two dimensional, normal to the slope and parallel to the slope.
3) The sloping boundary acts as a uniform area source of heat.
4) The pressure disturbance is negligible.
5) Steady state conditions are assumed.
6) The Prandtl solution requires a positive $\xi$ value, \( \frac{\Delta T}{\Delta z} T_0 \).

The solution applies only when the undisturbed temperature increases with height above the sloping surface as illustrated in Figure 1. Cases of decreasing temperature gradient with height fall under the treatment of stratified free convection within bounded layers. The solution breaks down when the inclination of the slope is zero.

Concerning the 5th assumption, since the oscillatory period is relatively long and since the slope velocities are high over the short distance of the radiometer, thermal equilibrium is readily reached via equation (1) during the slow oscillation and a steady state assumption appears to be valid.

**SOLUTION FOR VARIOUS INITIAL CONDITIONS**

The following four basic formulae Lettau, 1961 are solved:

Characteristic Length: \( L^* = 4 M^3 \sqrt{Pr} \xi P_\alpha \frac{2}{3n} \alpha \)  \( (2) \)

Temperature Disturbance at Slope: \( \Delta T_0 = L \left( T_0 \alpha \cos \alpha + G \rho \frac{Pr}{\mu C_P} \right) \)  \( (3) \)

Characteristic Velocity: \( U^* = \left( \Delta T_0 / T_0 \right) \left( g / \xi P_\alpha \right)^{1/2} \)  \( (4) \)

Effective Displacement Thickness: \( D^* = 1.65 L \)  \( (5) \)
From these solutions the dimensionless modules Reynolds number, $Re$, and Rayleigh number, $Ra$, were computed for various slopes, effective thickness, and heating rates, the latter being the main variable. Defining the Reynolds and Rayleigh numbers, respectively, as:

\[
Re = \rho \nu^* L/\mu^2 \tag{6}
\]
\[
Ra = g \delta L^4 \rho^3 C_p/\kappa \mu \tag{7}
\]

and considering equations (2) through (5), we can draw upon important proportionalities:

(1) For small slopes and constant heat exchange, $L \sim \sqrt[3]{\sin^2 \alpha}$

and, therefore

\[
Re \sim \sqrt[3]{\sin^2 \alpha} \sim L \tag{6a}
\]

(2) Normally, too,

\[
Ra \sim L^4 \tag{7a}
\]

Since we measure $\theta_o, T_o, \alpha$, and $\sqrt[n]{\nu}$ in the normal course of our test flight procedure we can readily enter these solutions 2 through 7a. An experiment was set up to investigate the effect of heating. Figure 2 illustrates a frame with the radiometer inside and a black radiating surface to provide source radiation at energy levels below (or above) that of the surrounding atmosphere's radiative equilibrium. The black surface is placed over the radiometer in the frame. Temperatures within the radiometer upper cells were monitored at the up-slope edge, center and down-slope edge (at 1 mm above the black radiometer sensor surface) for the various tilt angles and various forcing function variables, $\theta_o$. If we assert that Rayleigh modules, $Ra$, less than 1700 occur only in processes involving molecular conduction; that $Ra$ modules equal to 1700 measure the onset of stable cellular convection; and that $Ra$ modules exceeding 50,000 exist only for turbulences we can theoretically determine the existence
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or non-existence of turbulence, stable cellular convection or molecular conductivity. Furthermore such theoretical results may be compared with experimental results. Each division on the recorder scale, figure 3, represents 2.0°C. An examination of the scale in this figure corresponding to conditions in serial experiments 6 through 11, table 1, shows no displacement of the trace from the tilt condition to no tilt.

Table 1 is a summary of machine computed results of the Reynolds and Rayleigh modules together with the initial values of $\Theta_0, \alpha$ and $T_0$. Results appear as $L, D^*, Re$ and $Ra$. One should note that no restraint has been placed on "L" the characteristic length, in the machine computations. In practice, with a vertical cell dimension of 0.7 cm, it appears justified to use 0.35 cm as the height of the level of maximum wind speed, L.

Serial experiments 1 through 5 contain Rayleigh and Reynolds module solutions from observed data for a typical night time low level ascent portion of a radiometer flight.

Serial experiments 6 through 11, typical high altitude nocturnal conditions, also impose a much greater than normal heat exchange (0.0067 ly/sec) on the surface and hence, without restraint on L, the Rayleigh numbers exceed the critical value for the onset of stable cellular convection. If we impose the 0.35 cm restraint on L for the conditions of serial experiment no. 11 we obtain a Rayleigh number of 127, far below the critical value. Since all other initial conditions remained constant, equation (7a) was used to obtain the Rayleigh number. The new Reynolds number was computed from equation (6a).

Serial experiments 12 through 14 impose the extreme conditions of a late afternoon release under clear dry atmospheric conditions and high shelter temperatures. The heat exchange has been doubled over that of similar observed conditions. Serial experiment 13 exceeded the critical
Rayleigh module, but assumed a very low value, 25, when the restraint of the polyethylene cell (1/2 x .7 cm) dimension was imposed. The Reynolds number in serial 13 also dropped from 329 to 106.

Serial experiment 15 is a test of the effect of increasing the thickness of the double cell structure to 2.8 instead of 1.4 (2 x .7) centimeters. This results in an increase of both the Reynolds and Rayleigh modules.

Serial experiments 16 and 17 deal with a daytime case but with \( \frac{dT}{dz} > 0 \). It is for the lower polyethylene surface having an absorptivity of 2 percent. This results in a positive heat transfer to the polyethylene of 0.00307 ly/sec for \( \theta_0 \). The remaining radiation is transmitted through the polyethylene. The experiment was carried out with a tilt of 2° to obtain maximum Rayleigh modules. Without height restraint the Ra module reached 8.77 x 10^5. With restraint it dropped to 25.

Our final experiment was done with the same apparatus at 0° slope, and with \( \frac{dT}{dz} \) decreasing with distance above the black sensing surface, resulting in a study in stratified free convection in bounded layers, supplementary to the theory of thermal slope winds just discussed. Under conditions involving a temperature gradient of 60° per 1.4 cm and a surface (sensor) temperature of 360°A, daytime conditions, the Rayleigh module was 640, well below the critical value. This assumed a dimension between cells of 0.70 cm. This appears in table 1 as Serial Experiment 18. Of course, in this instance we cannot use 0.35 cm as the depth since we no longer are dealing with thermal slope winds. Thus it is obvious that we do not experience convection in the cell structure of the economical radiometer.
CONCLUSIONS

As long as the Rayleigh number does not exceed a numerical value of 1700, equilibrium is stable and any convection is damped out by the effects of viscosity and conduction. Of course, the value of the critical Rayleigh number depends upon the physical conditions of the experiment, that is the bounding surfaces if they exist. For two rigid boundaries, as in our case, this is approximately 1700.

The critical Reynolds module for the onset of stable cellular convection of our experiments is 1000. This was never exceeded in the experiments tried under tilt conditions.
REFERENCES


H. Lettau, 1961, manuscript, *Steady Flow Driven by Buoyancy Forces Which are Produced by Heat Exchange at a Sloping Boundary.*


LIST OF FIGURES

Figure 1: Thermal slope wind conditions

Figure 2: Economical radiometer and radiating surface in frame.

Figure 3: Portion of Leeds-Northrup Recorder record made by upper radiometer surface in level and tilted position under radiating surface.